Habits and Durability in Consumption, and the Effects of Exchange Rate Policies

by

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Abstract

A model in which consumption exhibits durability, and habits develop over the flow of services provided by them, is used to study the effects of exchange rate policies. The results of recent studies with regard to the effects of exchange rate policies are brought closer to some recent empirical findings. It is shown that, after a change in the rate of devaluation, the adjustment of consumption, real money holdings, and the country's net foreign asset position will very likely be non-monotonic.

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I. Introduction

In a recent paper, Mansoorian (1996a) uses the habit persistence model of Ryder and Heal (1973) in order to examine the effects of exchange rate policies for a small open economy. Recent findings (e.g., by Heaton (1995)) suggest that the empirical performance of the habit persistence model improves considerably when one extends it in order to allow for durability in consumption. The purpose of this paper is to extend the model used by Mansoorian to bring it close to these empirical findings; and to further enhance our understanding of the effects of government policies.

Mansoorian's paper builds on the works of Obstfeld (1981a, 1981b) who first considered the effects of exchange rate policies for a small open economy in an optimising framework. Obstfeld assumes that instantaneous utility is a function of consumption and real money holdings, as in Sidrauski (1967). Also as in Uzawa (1968) the rate of time preference is an increasing function of instantaneous utility. Mansoorian uses the habit persistence model instead of the Uzawa preferences, and shows that richer results can be obtained in that framework.

In the habit persistence model instantaneous utility depends on both current full consumption and the habitual standard of living, represented by a weighted average of past levels of full consumption.\(^1\) As in Obstfeld (1981a), Mansoorian assumes that the policy of the central bank is directed at having a constant rate of devaluation of the domestic currency. It was shown that an increase in the rate of devaluation requires a fall in the steady state habitual standard of living, if the nominal interest rate is initially positive. This is because in models

\(^1\) Full consumption is the utility derived from current consumption and current real money holdings.
with real balances in the utility function steady state utility falls with an increase in nominal interest rates. With the habit persistence model if preferences exhibit adjacent complementarity then after any shock the agent tries to maintain his habitual standard of living. In that case, after the increase in the rate of devaluation the representative agent will substitute present for future consumption by reducing his savings. Thus, short run consumption will rise. Short run money holdings will also not fall by as much as in the long run. The country will, therefore, run a current account deficit, and both consumption and real money holdings will fall along the adjustment path to the long run equilibrium. On the other hand, if preferences exhibit adjacent substitutability then the adjustments of the economy will be qualitatively the same as those predicted by Uzawa preferences. In that case, an increase in the rate of devaluation leads to a current account surplus. Both consumption and real money holdings fall in the short run. Then both of these variables increase along the adjustment path to the new steady state equilibrium.2

Recent empirical findings indicate that extensions of the habit persistence model to allow for durability in consumption, with habits developing over the flow of services provided by such durables, perform very well with regard to the predictions of the consumption based asset pricing literature (see, for example, Heaton).3 In these models durability tends to make consumption in adjacent dates substitutable, while habits are assumed to make them complementary. Heaton finds that the durability effects are dominant over a period of four months; but they are dominated by the habit effects after that. He concludes that "habit

2 With perfectly flexible prices a once and for all devaluation will have no real effects.

3 Such a model has also been used by Mansoorian (1996b) in order to study the well known J-curve phenomenon.
persistence substantially improve the model’s ability to fit stock and bond returns *only if local substitution is also present*” (p. 683).

The purpose of the present paper is to extend the model used in Mansoorian (1996a) in order to allow for durability in consumption, and bring it closer in line with these empirical findings. As in Obstfeld and Mansoorian, with no price rigidities a once and for all devaluation will have no real effects. As in Mansoorian, an increase in the rate of depreciation of the domestic currency will decrease the steady state habitual standard of living if the nominal interest rate is initially positive. To sustain these lower standards the steady state stock of durables should also fall. If the durability effects are dominant in the short run and habit effects in the long run then immediately after an increase in the rate of devaluation there will be a fall in both consumption and real money holdings, and the country will run a current account surplus. After this both consumption and real money holdings will increase during the adjustment process. However, there will come a time at which the habit effects will become dominant. At that point the country will start to run a current account deficit, and consumption and real money holdings will start to fall until the new steady state is reached.4

Thus, with the simultaneous presence of habits and durability the adjustment of consumption, real money holdings, and the country’s net foreign asset position will be non-monotonic. In particular, during the adjustment process these variables may overshoot their steady state levels. Such overshooting is in contrast to the well known overshooting results in

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4 On the other hand, Mansoorian (1996b) shows that for such a model to be consistent with the empirical findings of Backus, Kehoe and Kydland (1994), regarding the J-curve, the habit effects should be dominant in the short run and durability effects in the long run. The adjustment of the relevant variables for that case will be correspondingly different.
the spirit of Dornbusch (1976). There a variable is said to overshoot its long run level if its response immediately after a policy change is larger than its long run response. Overshooting in the present paper, on the other hand, does not need to occur immediately after a policy change, but it will occur somewhere along the adjustment process to the long run equilibrium.

The paper is organized as follows. The problems of the representative agent and the government are set out in section II. The perfect foresight path is derived in section III. The effects of exchange rate policies are considered in section IV. Some concluding remarks are made in section V.

II. The Model

The model corresponds most closely to Obstfeld's (1981a), and Mansoorian's. The foreign currency price of the single good in the model is fixed at \( P^* \). The economy is small and takes \( P^* \) as given. The domestic currency price of this good is \( P = EP^* \), where \( E \) is the exchange rate (the price of foreign currency in terms of domestic currency). The rate of inflation is equal to the rate of depreciation of the domestic currency (\( E/E \)), which is denoted by \( \dot{E}/E \).

The objective function of the representative agent is a variant of that used by Heaton; and it extends the one used by Mansoorian:

\[
\int_0^T e^{\delta t} U(s_t, \%T_t, h_t) \, dt,
\]

where \( \delta \) is the rate of time preference, and for any time \( t \) \( T_t \) is a utility measure of the agent's current consumption and real money holdings. It is defined as

\[
T_t = T(c_t, m_t),
\]

where \( T(\Theta) \) is a homothetic subutility function.
s_t is the stock of durables at time t which are inherited from the past. We model it as a weighted sum of \( T_j \) (\( j < t \)), with exponentially declining weights given to more distant values of \( T_j \):

\[
s_t = \sum_{j < t} \frac{e^{-*(j-t)}}{\Omega \Delta} w_j \]

where \(* > 0\).

Strictly speaking, \( s_t \) should be a weighted sum of past levels of \( c \) (not \( T \)). However, if \( s_t \) is modelled that way then the marginal rate of substitution between \( c \) and \( m \) at any date will depend on the past levels of \( c \). This will prevent us from using the standard simple two stage programming approach for solving dynamic models with two goods, complicating the analysis without substantially changing the results. Moreover, modelling \( s_t \) as a weighted sum of past levels of \( T \) enables us to draw heavily on the results already available from Mansoorian.

Note that \( s_t + T_t \) is the total services of the durable goods that are enjoyed at time t.

Also, from (3) it follows that the evolution of \( s_t \) is given by

\[
s_t' = \sum_{j < t} \frac{e^{-*(j-t)}}{\Omega \Delta} w_j
\]

\( h_t \) is the habitual standard of living at time t. These habits are developed over the flow of past consumption services. Thus, \( h_t \) is a weighted sum of \( (s_j + T_j) \) (\( j < t \)), with exponentially declining weights given to more distant values of \( s_j + T_j \):

\[
h_t = \sum_{j < t} \frac{e^{-D(j-t)}}{\Delta \Omega} \left[ s_j + T_j \right]
\]

where \( D > 0 \). From (5) it follows that the evolution of \( h_t \) is given by

\[
h_t' = \sum_{j < t} \frac{e^{-D(j-t)}}{\Delta \Omega} \left[ s_j + T_j \right]
\]
We maintain assumption (P.1)! (P.5) of Ryder and Heal (pp. 2-3), regarding the
momentary utility function, U(\theta). Thus, momentary utility is assumed to be: (P.1) increasing
in the current flow of services consumed, U_t > 0; (P.2) non-increasing in habits, U_2 \neq 0; (P.3)
increasing in uniformly maintained T, i.e., U_t(x, x) + U_2(x, x) > 0 for all x > 0; (P.4) concave
in its two arguments; and (P.5) \lim_{x \to 0} U_1(x, h)' = 4 and \lim_{x \to 0} [U_1(x, x) + U_2(x, x)] = 4.

The agent is endowed with y units of the good at any time t. He also receives real
transfers of J_t from the government. There are two kinds of assets in the model, money
balances and internationally traded bonds whose foreign currency price is fixed, and which
have a fixed rate of return of r. The real assets of the representative agent are

\[ a_t', m_t, \theta_t, \]  

(7)

where \theta_t is his bond holdings.

His flow budget constraint is

\[ \dot{a}_t' + y + a_t \% J_t + c_t \% (r \% m_t) = 0. \]  

(8)

Finally, the agent should satisfy the intertemporal solvency condition

\[ \lim_{t \to 64} e^{r_1 a_t} = 0. \]  

(9)

Thus, the representative agent's problem is to maximize (1), subject to (4), (6)! (9), and
the initial conditions (i.e., h_0, s_0, and a_0), taking the paths of \{J_t\}, and \{r_t\} as given. Along a
perfect foresight path, the agent's expectations about \{J_t\} and \{r_t\} coincide with the actual
paths of these variables.
As in Obstfeld (1981a), we assume that government consumption is totally unproductive, and that monetary policy is directed at maintaining a constant rate of inflation (that is, a constant rate of depreciation of the domestic currency). The government chooses $J_t$ in order to satisfy its flow constraint, which says that it should finance its expenditures $(g+J_t)$ from the interest on its bond holdings and the inflation tax:

$$J_t % g + rR_t % m_t,$$  \hspace{1cm} (10)

where $R_t$ is the central bank's foreign reserves at time $t$.

### III. The Perfect Foresight Path

In this section we derive the perfect foresight path of the model. To do so we first solve the representative agent’s problem. Then we consider the country’s external adjustment along the equilibrium path in light of the assumptions we have made about government policies.

Because $T(\theta)$ is homothetic, and also the marginal rate of substitution between $c_t$ and $m_t$ at any time $t$ is independent of consumption and real money holdings at other dates, the problem of the representative agent can be solved in two stages. In the first stage, for a given level of expenditures $Z_t$, maximize $T(c_t, m_t)$ subject to $Z_t = c_t + (r + \delta)m_t$ to obtain the indirect utility function $Z_tV(r + \delta)$. In the second stage choose $\{Z_t\}$ so as to

$$\max_{m} 4U(s_t, \%Z_tV(r), h_t)) e^{\delta t} dt,$$

s. t. $\dot{a}_t = y % r % a_t % j \& Z_t$, \hspace{1cm} (4), (6), (9), and the initial conditions.

The Hamiltonian for his problem is

$$H' = U(s_t, \%Z_tV, h_t) \% N_t [Z_tV \& s_t] \% \delta [\{\delta s_t, \%Z_tV \& h_t\}] \% q_t [ra_t, %y % j, \& Z_t],$$

where $N_t$, $\delta$, and $\dot{a}_t$ are the shadow prices of $s_t$, $h_t$ and $a_t$, respectively.
The optimality conditions are:

\[ H_Z / U_1 V \% NV \% BDV & \mu' 0, \]  
\[ \& H_s \% 2 N / \& U_1 \% N* \& 8D \% 2 N' \& N, \]  
\[ \& H_n \% 2 8 / \& U_2 \% 8D \% 2 8' \& \delta, \]  
\[ \& H_n \% 2 \mu / \& \mu, \& 2 \mu' \& \mu, \]  

and the standard transversality conditions.

From (14) it is clear that a steady state can be reached only if

\[ r = 2. \]  

This is a standard assumption that is made in the literature, and we will maintain it from now on. From (14) and (15) it follows that \( \dot{\mu} = 0 \), and that \( \hat{\gamma} \) is always at its steady state level.

Linearizing (11) around the steady state, using the fact that \( \dot{\mu} = 0 \), we obtain

\[ (Z_t \& \overline{Z})' \& \frac{1}{V} (s_t \& \overline{s}) \& \frac{U_{12}}{U_{11}} V (h_t \& \overline{h}) \& \frac{D}{U_{11}} V (\delta_t \& \overline{\delta}) \& \frac{1}{U_{11}} V (N_t \& \overline{N}), \]  

where bars over variables denote steady state values.

Linearizing (6), (12) and (13) around the steady state, using (15) and (16), we obtain

\[
\begin{bmatrix}
\hat{h}_t \\
\hat{\delta}_t \\
\hat{N}_t
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
0 & 0 & a_{33}
\end{bmatrix} \begin{bmatrix}
h_t \& \overline{h} \\
\delta_t \& \overline{\delta} \\
N_t \& \overline{N}
\end{bmatrix},
\]  

where,

\[ a_{11} = \frac{\& D(U_{12} \% U_{11})}{U_{11}} \]
In the differential equation system (17) \( h \) is a state variable, while \( \theta \) and \( N \) are both jump variables. Therefore, for saddlepoint stability of the system the coefficient matrix should have two positive and one negative eigenvalues. For this we need \( a_{11} a_{12} \neq a_{12} a_{21} < 0 \), which, given our assumptions (P.1)-(P.4), could be easily satisfied. If \( > \) is the negative eigenvalue of the coefficient matrix in (17) that is,

\[
> \frac{r \sqrt{r^2 + 4(a_{11} a_{22} - a_{21} a_{12})}}{2},
\]

then the stable path to the steady state will be given by:

\[
(h, \bar{h})' = (h_0, \bar{h}) e^{\lambda t}, \tag{18}
\]

\[
(\theta, \bar{\theta})' = \frac{> a_{11}}{a_{12}} (h_0, \bar{h}) e^{\lambda t}, \tag{19}
\]

\[
(N, \bar{N})' = 0. \tag{20}
\]
To obtain the solution for \( s_t \), linearize (4) around the steady state, and then use (16) and (18) to get

\[
\dot{s}_t = (1 \%\%) (s_t \& s) \& S (h_0 \& h) e^{\gamma}.
\]

(21)

where

\[
S = \begin{bmatrix}
U_{12} & D
U_{11} & a_{11}
\end{bmatrix} = (1 \%\%) D.
\]

The solution to (21) is

\[
s_t = \frac{S}{1 \%\%} (h_0 \& h) e^{\gamma} \left[ \frac{S}{1 \%\%} (h_0 \& h) \% (s_0 \& s) \right] e^{(1 \%\%) t}.
\]

(22)

To determine the country's external adjustment along the perfect foresight path first note that (8) and (10) imply that

\[
\dot{f}_t = y \% f_t \& c_t \& g,
\]

(23)

where \( f_t \) is the country's net foreign asset position \( (b_t + R_t) \).

To characterize the evolution of \( f_t \) along the perfect foresight path, first note that by Roy's identity \( m_t \& Z_t V^{-1} V \). Thus,

\[
c_t = Z_t \left[ 1 \% (r \% \%) V^{-1} \right].
\]

(24)

Substituting for \( c_t \) from (24) into (23), we get

\[
\dot{f}_t = y \% f_t \& g \& Z_t \left[ 1 \% (r \% \%) V^{-1} \right].
\]

(25)

Linearizing this equation around the steady state, and then using (16), (18) to (20), and (22), we obtain
The solution to this differential equation is

\[ f_t \in \bar{f} \% 7_1 e^{\%} 7_2 e^{\%}(f_0 \& \bar{f}) \& 7_1 \& 7_2 e^{\%}, \quad (26) \]

where

\[ 7_1 \% S(\% 7_1 e^{\%}) \% [1 \% (r \% a) V/V] \% (h_0 \& \bar{h}), \quad (27) \]

and

\[ 7_2 \% S \% [1 \% (r \% a) V/V] \% (h_0 \& \bar{h}) \& [1 \% (r \% a) V/V] \% (s_0 \& \bar{s}). \quad (28) \]

Clearly, for (26) to converge we will need

\[ [(f_0 \& \bar{f}) \& 7_1 \& 7_2]' = 0, \quad (29) \]

which, for given values of \( f_0, h_0 \) and \( s_0 \), shows how \( \bar{f}, \bar{h} \) and \( \bar{s} \) should be related for saddlepoint stability. With this condition, (26) reduces to

\[ f_t \& \bar{f} \% \bar{7}_1 e^{\%} \% 7_2 e^{\%}(f_0 \& \bar{f}) \% 7_1 \& 7_2 e^{\%} \% (1\% \%\% a). \quad (30) \]

Equations (18)\% (20), (22) and (30) give us the stable path of the model to the steady state equilibrium.

**III. The Effects of Exchange Rate Policies**

In this section we examine the effects of exchange rate policies. First consider the steady state of the model, which is characterised by equation (11), and by equations (4), (6), (12), (13), and (25), with \( s = h = f = N = B = 0 \). These are six equations in seven unknowns: \( \bar{s}, \bar{h}, \bar{f}, \bar{Z}, \)
The seventh equation is obtained from (29), which gives us the following relationship for the changes in the steady state levels of the state variables, $f$, $s$, and $h$:

$$\frac{df}{d\bar{r}}, \frac{ds}{d\bar{r}}, \frac{dh}{d\bar{r}}, \frac{Z(\bar{r} + \bar{\alpha})}{(1 + \bar{\alpha})} \left(1 + \frac{1}{V} \right) \frac{\bar{S} \bar{r}}{\bar{\alpha} \bar{r}} \neq 0. \tag{32}$$

The numerator of the right hand side of (32) is $(\bar{r} + \bar{\alpha})(1 + \bar{\alpha})\bar{e}_{11}$, where $\bar{e}(\bar{r} + \bar{\alpha}, \bar{T})$ is the expenditure function corresponding to $T(c, m)$ at the initial steady state (see Mansoorian, footnote 6). Thus, the numerator of the right hand side of (32) is negative as long as the nominal interest rate $(\bar{r} + \bar{\alpha})$ is positive, while the denominator is positive. With money in the utility function, steady state level of utility is maximized when nominal interest rates are zero. Hence, if initially the nominal interest rates are positive then an increase in the inflation rate $(\bar{r})$ will reduce the steady state habitual standard of living, and the steady state level of the stock of habits that are required to sustain these standards.

To understand the resulting adjustment of $f$, $c$ and $m$ with both habits and durability it will be instructive to first consider the adjustment of these variables in the presence of each one of these effects in isolation. The effects of an increase in $\bar{r}$, in the presence of adjacent
complementarity alone are fully worked out in Mansoorian, when condition (23) in that paper is met. The increase in the inflation rate will immediately reduce the real money holdings. With adjacent complementarity the representative individual will want to maintain the habitual standard of living he inherited from the past. Hence, after the increase in, savings will fall, and the country will run a current account deficit. The country's net foreign asset position will then decline gradually over time. As a result c will be declining over time. With homothetic preferences, m and c will be held in the same ratio during the adjustment period. Hence, m will also be falling over time. The adjustment of c and m for this case are shown in Figures (1) and (2) in Mansoorian.

Now consider the adjustment of the model after an increase in, with durability, and no habits. Then preferences will exhibit adjacent substitutability. In this case, too, the increase in, will reduce m immediately. However, in this case after the increase in, there will be an increase in savings and a current account surplus. The reason for this is as follows. The representative agent has a relatively large stock of durables when he experiences a decline in his permanent real income. He, moreover, knows that his steady state stock of durable goods should fall to accommodate the decline in his permanent real income. It is optimal for him to consume the services from the relatively large stock of durables that he has inherited from the past, and to save in order to be able to replace them when their stock has gone down by the appropriate amount. Hence, in that case, after the increase in, there is a fall in aggregate

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5 The adjustment of the model in this case will be the same as in Mansoorian when preferences exhibit adjacent substitutability (i.e., when condition (23) in that paper does not hold), with the stock of durables in the present model (s) replacing habits in that model (which were denoted by S). (A sufficient condition for adjacent substitutability in Mansoorian is that an increase in S reduces the marginal utility of real consumption! i.e., U_{12}<0 in that paper.)
expenditures and a current account surplus. The country’s net foreign asset position will then improve gradually over time. As a result, c and m will be increasing during the adjustment process, as shown in Figures (3) and (4) in Mansoorian.

Heaton finds evidence in favour of the durability effects being dominant in the short run (for a period of four months), and the habit effects in the long run. His findings suggest that after the increase in , there will be an improvement in the country’s net foreign asset position, which will then be followed by a deterioration. Also, from figures (1)! (4) in Mansoorian we can infer that the adjustment of c and m will be as shown in Figures (1) and (2) below. Thus, after the increase in , there will be a fall in c and m, as in the short run the durability effects are dominant. Then over time both of these variables will be rising until there comes the time at which the habit effects become dominant. At that point c and m start to fall until we reach the new steady state.6

Of course, with the simultaneous presence of both habits and durability the interaction between the two effects will add an additional twist to the dynamics of the model. But this will not alter the general properties of the adjustment process.

Recall that $c_t = Z_t [1 + (r + , )VN/V]$. Hence, from (16), (18)! (20) and (22) we obtain the adjustment of c along the stable path:

$$c_t \Rightarrow 7_2 (r \Rightarrow e^{\gamma (1 + \sigma) t} e^{\delta(1 + \sigma) t}. \quad (33)$$
The negative eigenvalue \( \lambda \) in the present model is the same as the eigenvalue in Mansoorian (p. 123), where \((c_t - \bar{c}) \) was proportional to \( e^{\lambda t} \). Hence, the presence of the second term on the right hand side of equation (33) in the present paper is due to durability. Moreover,

\[
\dot{c}_t \succ (r \& \succ \gamma_{1} e^{\lambda (1 + \delta + \phi) t} \& (1 + \phi) \gamma_{2} e^{\delta (1 + \phi) t}.
\]

If we regard the first term on the right hand side of this equation to be due to habits, and the second term due to durability, then we will need \( \gamma_1 > 0 \) and \( \gamma_2 < 0 \) in order for the habit effects on \( \dot{c}_t \) to be negative, and the durability effects positive. Heaton's empirical findings suggest that \( \dot{c}_0 > 0 \). For this we need \((1 + \phi)(1 + \delta + \phi) \gamma_2 < (r \& \succ \gamma_1 \& \gamma_1) \). Finally, for the habit effects to be dominant eventually (i.e., \( \dot{c}_t < 0 \) when \( t \) is sufficiently large) we will need \((1 + \phi) \) to be much larger than \( \succ \). With these assumptions we will have the non-monotonic adjustment of \( c \) (and \( m \)) that we discussed above.

Backus, Kehoe, and Kydland (1994) document evidence from various countries which suggest that the trade balance is negatively correlated with current and future movements in the terms of trade, but positively correlated with past movements. In a recent paper, Mansoorian (1996b) shows that Backus et al.’s findings indicate that the habit effects should be dominant in the short run, and the durability effects in the long run. These findings suggest that after an increase in \( c \), the country’s net foreign asset position will deteriorate; but eventually it will start to improve. In that case, immediately after the increase in \( c \), \( c \) will increase and \( m \) will fall. Then both of these variables will be falling over time, until there comes the time at which the durability effects become dominant. At that point \( c \) and \( m \) will start to increase until we reach

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7 To see this, substitute from equations (12) and (13) into (11) in that paper.
the new steady state equilibrium. The adjustment of these variables for this case are shown in figures (3) and (4) below.

IV. Conclusions

Recent empirical findings suggest that it may be important to allow for durability in consumption when using the habit persistence model. In this paper we have discussed the implications of such a model with regard to exchange rate policies. It was shown that in such a model the adjustment of consumption, real money holdings and the country's net foreign asset position after a change in the rate of devaluation will very likely be non-monotonic.

In particular, Heaton's empirical findings suggest that the durability effects should be dominant in the short run and the habit effects in the long run. In such a case after an increase in the rate of devaluation of the domestic currency there will be a fall in both consumption and real money holdings, and a current account surplus. After this the country's foreign asset position will improve, and consumption and real money holdings will increase along the adjustment path, until there comes the time at which the habit effects become dominant. At that point the country's net foreign asset position will start to deteriorate, and consumption and real money holdings will start to fall until the new steady state is reached.
References


